HH 1.14

$$E = \int_0^t VI\,dt$$

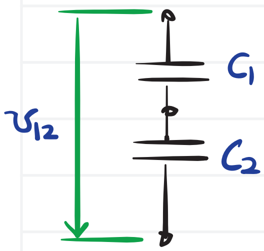
$$= \int_0^t VC\frac{dV}{dt}\,dt$$

$$= C\int_{V(0)}^{V_f} V\,dV$$

$$= \frac{1}{2}C\left(V_f^2 - V(0)^2\right)$$

$$= \frac{1}{2}CV_f^2$$

HH 1.15



KCL: $i_{c_1} = i_{c_2} = i$

Elemental: $i_{c_1} = C_1 \dfrac{dv_{c_1}}{dt}$ $\qquad i_{c_2} = C_2 \dfrac{dv_{c_2}}{dt}$

$$\Longrightarrow \quad v_{c_1} = \frac{1}{C_1}\int_0^t i \, dt \qquad v_{c_2} = \frac{1}{C_2}\int_0^t i \, dt$$

KVL: $v_{12} = v_{c_1} + v_{c_2}$

$$= \frac{1}{C_1}\int_0^t i \, dt + \frac{1}{C_2}\int_0^t i \, dt$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i \, dt$$

$$\Longrightarrow \quad \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} v_{12} = \int_0^t i \, dt$$

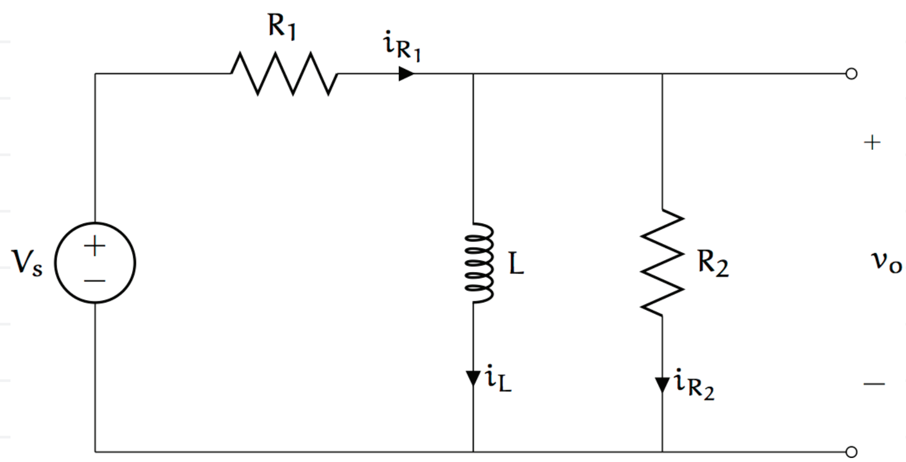
$$\Longrightarrow \quad \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \frac{dv_{12}}{dt} = i \; .$$

This is in the form of an elemental eq. for a capacitor with capacitance

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \; . \qquad \text{QED}$$

From Assignment #3

For the circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_s(t) = A \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $i_L(t)|_{t=0} = 0$ be the initial inductor current. Hint: you will need to solve a differential equation for $i_L(t)$.



1. Signs. Already given

2. El. eq. $v_{R_1} = i_{R_1} R_1 \qquad v_{R_2} = i_{R_2} R_2 \qquad v_L = L \frac{di_L}{dt}$

3. KCL. $i_{R_1} = i_L + i_{R_2}$

4. KVL. $V_s = v_{R_1} + v_L \qquad v_L = v_{R_2} \qquad v_o = v_{R_2}$

5. Algebra.

$$\frac{di_L}{dt} = \frac{1}{L} v_L = \frac{1}{L} v_{R_2} = \frac{R_2}{L} i_{R_2} = \frac{R_2}{L}(i_{R_1} - i_L)$$

$$= \frac{R_2}{L}\left(\frac{v_{R_1}}{R_1} - i_L\right)$$

$$= \frac{R_2}{L}\left(\frac{1}{R_1}(V_s - v_L) - i_L\right)$$

$$= \frac{R_2}{L}\left(\frac{1}{R_1}\left(V_s - L\frac{di_L}{dt}\right) - i_L\right) \Rightarrow$$

$$\underbrace{\left(\frac{L}{R_2} + \frac{L}{R_1}\right)}_{\tau} \frac{di_L}{dt} + i_L = \frac{1}{R_1} V_s .$$

6. DE. Solve DE for $i_L(t)$. Then $v_o(t) = v_L(t) = L\frac{di_L}{dt}$.