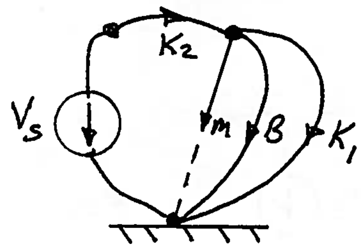
5-2

PROBLEM 5.2

(a) SYSTEM a



(b) ORDER = 3

St. VAR.: V_m, F_{K_1}, F_{K_2}

(c) $\frac{dV_m}{dt} = \frac{1}{m}\left[F_m\right]$

$\frac{dF_{K_1}}{dt} = K_1 \; V_{K_1}$

$\frac{dF_{K_2}}{dt} = K_2 \; V_{K_2}$

$F_B = B \, V_B$

$F_m = F_{K_2} - F_B - F_{K_1}$

$V_{K_1} = V_m$

$V_{K_2} = V_s - V_m$

$V_B = V_m$

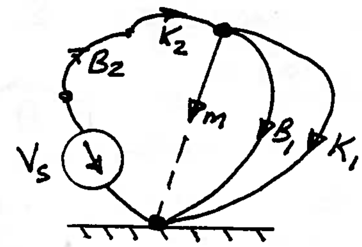
The 3 st. EQ.

$$\frac{dV_m}{dt} = \frac{1}{m}\left[F_{K_2} - F_{K_1} - BV_m\right]$$

$$\frac{dF_{K_1}}{dt} = K_1 \, V_m$$

$$\frac{dF_{K_2}}{dt} = K_2\left[V_s - V_m\right]$$

SYSTEM b



ORDER = 3

St. VAR: V_m, F_{K_1}, F_{K_2}

$\frac{dV_m}{dt} = \frac{1}{m}\left[F_m\right]$

$\frac{dF_{K1}}{dt} = K_1 \; V_{K1}$

$\frac{dF_{K2}}{dt} = K_2 \, V_{K2}$

$F_{B_1} = B_1 \, V_{B_1}$

$V_{B_2} = \frac{1}{B_2} \, F_{B_2}$

$F_m = F_{K_2} - F_{B_1} - F_{K_1}$

$V_{K_1} = V_m$

$V_{K2} = V_s - V_{B2} - V_m$

$V_{B_1} = V_m$

$F_{B_2} = F_{K_2}$

The 3 st. EQ.

$$\frac{dV_m}{dt} = \frac{1}{m}\left[F_{K_2} - B_1 V_m - F_{K_1}\right]$$

$$\frac{dF_{K_1}}{dt} = K_1\left[V_m\right]$$

$$\frac{dF_{K_2}}{dt} = K_2\left[V_s - \frac{F_{K2}}{B_2} - V_m\right]$$

PROBLEM 5.2 CONTINUED

$$\frac{d}{dt}\begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix} = \begin{bmatrix} -\frac{B}{m} & -\frac{1}{m} & \frac{1}{m} \\ K_1 & 0 & 0 \\ -K_2 & 0 & 0 \end{bmatrix} \bullet \begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_2 \end{bmatrix} V_s$$

$$\frac{d}{dt}\begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix} = \begin{bmatrix} -\frac{B_1}{M} & -\frac{1}{M} & \frac{1}{m} \\ K_1 & 0 & 0 \\ -K_2 & 0 & -\frac{K_2}{B_2} \end{bmatrix} \bullet \begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_2 \end{bmatrix} V_s$$

(d)

OUTPUT = $Y = V_m$

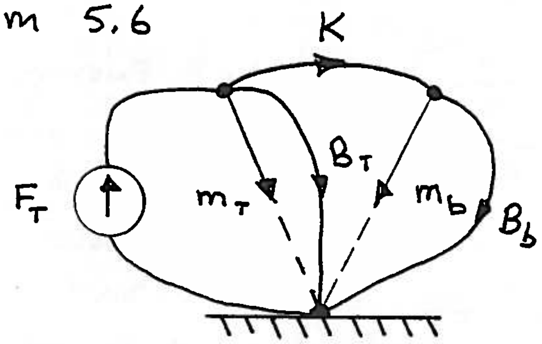
$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix}$$

OUTPUT = $Y = V_m$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_m \\ F_{K_1} \\ F_{K_2} \end{bmatrix}$$

5-8

PROBLEM 5.6



STATE VAR.
V_{m_T}, V_{m_b}, F_K

$(\,)_T$ = Tug Boat
$(\,)_b$ = barge
K: elastic cable

$$\frac{dV_{m_T}}{dt} = \frac{1}{m_T} F_{m_T} \qquad F_{m_T} = F_T - F_K - F_{B_T}$$

$$\frac{dV_{m_b}}{dt} = \frac{1}{m_b} F_{m_b} \qquad F_{m_b} = F_K - F_{B_b}$$

$$\frac{dF_K}{dt} = K\, V_K \qquad V_K = V_{m_T} - V_{m_b}$$

$$F_{B_T} = B_T V_{B_T} \qquad V_{B_T} = V_{m_T}$$

$$F_{B_b} = B_b V_{B_b} \qquad V_{B_b} = V_{m_b}$$

$$\frac{dV_{m_T}}{dt} = \frac{1}{m_T}\left[F_T - F_K - B_T V_{m_T}\right]$$

$$\frac{dV_{m_b}}{dt} = \frac{1}{m_b}\left[F_K - B_b V_{m_b}\right]$$

$$\frac{dF_K}{dt} = K\left[V_{m_T} - V_{m_b}\right]$$

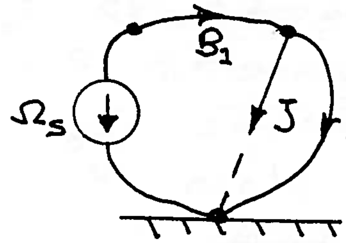
$$\frac{d}{dt}\begin{bmatrix} V_{m_T} \\ V_{m_b} \\ F_K \end{bmatrix} = \begin{bmatrix} -B_T/m_T & 0 & -1/m_T \\ 0 & -B_b/m_b & 1/m_b \\ K & -K & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{m_T} \\ V_{m_b} \\ F_K \end{bmatrix} + \begin{bmatrix} 1/m_T \\ 0 \\ 0 \end{bmatrix} F_T$$

$$y = V_{m_b} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{m_T} \\ V_{m_b} \\ F_K \end{bmatrix}$$

PROBLEM 5.8

SYSTEM (a)

(a)



(b)

$$\frac{d\Omega_J}{dt} = \frac{1}{J} T_J \qquad T_J = T_{B_1} - T_{B_2}$$

$$T_{B_1} = B_1 \Omega_{B_1} \qquad \Omega_{B_1} = \Omega_s - \Omega_J$$

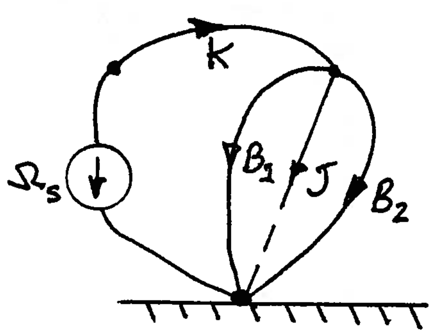
$$T_{B_2} = B_2 \Omega_{B_2} \qquad \Omega_{B_2} = \Omega_J$$

$$\frac{d\Omega_J}{dt} = \left[\frac{-(B_1 + B_2)}{J} \right] \Omega_J + \left[\frac{B_1}{J} \right] \Omega_s$$

(c) OUTPUT $= y = [1]\,[\Omega_J]$

SYSTEM (b)

(a)



STATE VAR.

Ω_J , T_K

PROBLEM 5.8 CONTINUED

(b)

$$\frac{d\Omega_J}{dt} = \frac{1}{J} T_J \qquad T_J = T_K - T_{B_1} - T_{B_2}$$

$$\frac{dT_K}{dt} = K \Omega_K \qquad \Omega_K = \Omega_S - \Omega_J$$

$$T_{B_1} = B_1 \Omega_{B_1} \qquad \Omega_{B_1} = \Omega_J$$

$$T_{B_2} = B_2 \Omega_{B_2} \qquad \Omega_{B_2} = \Omega_J$$

$$\frac{d\Omega_J}{dt} = \frac{-1}{J}(B_1 + B_2)\Omega_J + \frac{1}{J} T_K$$

$$\frac{dT_K}{dt} = -K\Omega_J + K\Omega_S$$

$$\frac{d}{dt}\begin{bmatrix}\Omega_J \\ T_K\end{bmatrix} = \begin{bmatrix} \frac{-(B_1+B_2)}{J} & \frac{1}{J} \\ -K & 0 \end{bmatrix} \cdot \begin{bmatrix}\Omega_J \\ T_K\end{bmatrix} + \begin{bmatrix}0 \\ K\end{bmatrix}\Omega_S$$

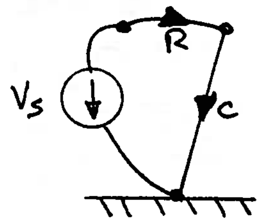
(c)

$$y = [1 \quad 0]\begin{bmatrix}\Omega_J \\ T_K\end{bmatrix}$$

5-14

PROBLEM 5.11

SYSTEM (a)

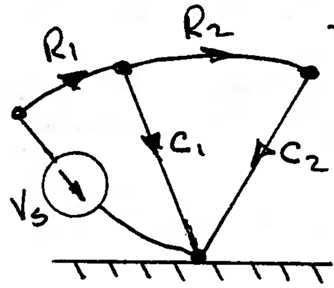


$$\frac{dV_c}{dt} = \frac{1}{C}\, i_c \qquad i_c = i_R$$

$$i_R = V_R/R \qquad V_R = V_s - V_c$$

$$\frac{dV_c}{dt} = \left[\frac{-1}{RC}\right] V_c + \left[\frac{1}{RC}\right] V_s$$

SYSTEM (b)



$$\frac{dV_{c_1}}{dt} = \frac{1}{C_1}\, i_{c_1} \qquad i_{c_1} = i_{R_1} - i_{R_2}$$

$$\frac{dV_{c_2}}{dt} = \frac{1}{C_2}\, i_{c_2} \qquad i_{c_2} = i_{R_2}$$

$$i_{R_1} = V_{R_1}/R_1 \qquad V_{R_1} = V_s - V_{c_1}$$

$$i_{R_2} = V_{R_2}/R_2 \qquad V_{R_2} = V_{c_1} - V_{c_2}$$

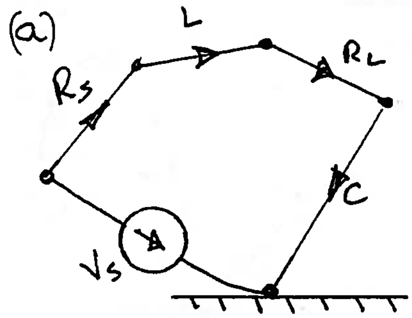
$$\frac{d}{dt}\begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} = \begin{bmatrix} \left(-1/R_1C_1 + 1/R_2C_1\right) & 1/R_2C_1 \\ 1/C_2R_2 & -1/R_2C_2 \end{bmatrix} \cdot \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} 1/R_1C_1 \\ 0 \end{bmatrix} V_s$$

$$y = V_{c_2} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix}$$

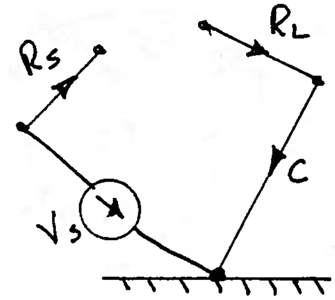
PROBLEM 5.12

SYSTEM (a)

(a)



LINEAR GRAPH



NORMAL TREE

(b)

$$\frac{dv_c}{dt} = \frac{1}{C} i_c \qquad i_c = i_L$$

$$\frac{di_L}{dt} = \frac{1}{L} V_L \qquad V_L = V_s - v_{R_s} - v_{R_L} - v_c$$

$$V_{R_s} = R_s i_{R_s} \qquad i_{R_s} = i_L$$

$$V_{R_L} = R_L i_{R_L} \qquad i_{R_L} = i_L$$

$$\frac{d}{dt}\begin{bmatrix} V_c \\ i_L \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -\frac{(R_s + R_L)}{L} \end{bmatrix} \cdot \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s$$

(c)

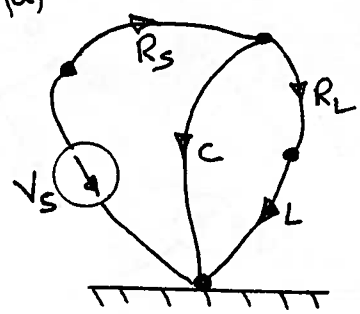
$$y = V_{OUT} = V_c + V_{R_L} + V_L = V_s - V_{R_s} = V_s - R_s i_L$$

$$y = \begin{bmatrix} 0 & -R_s \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + [1]\,[V_s]$$

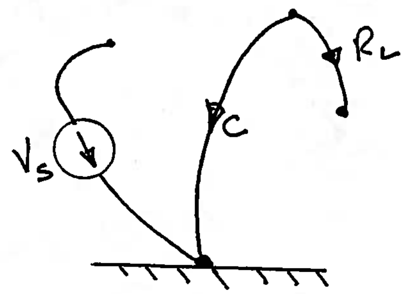
PROBLEM 5.12 CONTINUED

SYSTEM (b)

(a)



LINEAR GRAPH



NORMAL TREE

(b)

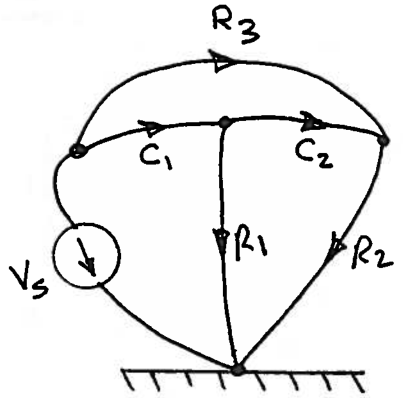
$\frac{dV_C}{dt} = \frac{1}{C} i_C$	$i_C = i_{R_S} - i_L$
$\frac{di_L}{dt} = \frac{1}{L} V_L$	$V_L = V_C - V_{R_L}$
$V_{R_L} = R_L i_{R_L}$	$i_{R_L} = i_L$
$i_{R_S} = \frac{1}{R_S} V_{R_S}$	$V_{R_S} = V_S - V_C$

$$\frac{d}{dt}\begin{bmatrix} V_C \\ i_L \end{bmatrix} = \begin{bmatrix} -1/CR_S & -1/C \\ 1/L & -R_L/L \end{bmatrix} \cdot \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 1/CR_S \\ 0 \end{bmatrix} V_S$$

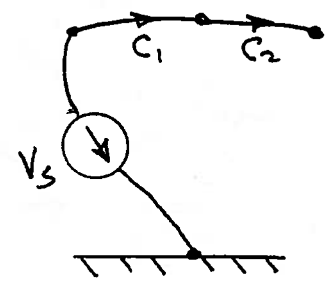
(c) OUTPUT $= y = V_C$

$$y = [1 \; 0] \begin{bmatrix} V_C \\ i_L \end{bmatrix}$$

PROBLEM 5.13



LINEAR GRAPH



NORMAL TREE

$\frac{dV_{c_1}}{dt} = \frac{1}{C_1} i_{c_1}$	$i_{c_1} = i_{R_1} + i_{R_2} - i_{R_3}$
$\frac{dV_{c_2}}{dt} = \frac{1}{C_2} i_{c_2}$	$i_{c_2} = i_{R_2} - i_{R_3}$
$i_{R_1} = \frac{1}{R_1} V_{R_1}$	$V_{R_1} = V_s - V_{c_1}$
$i_{R_2} = \frac{1}{R_2} V_{R_2}$	$V_{R_2} = V_s - V_{c_1} - V_{c_2}$
$i_{R_3} = \frac{1}{R_3} V_{R_3}$	$V_{R_3} = V_{c_1} + V_{c_2}$

$$\frac{dV_{c_1}}{dt} = \frac{1}{C_1}\left[\frac{V_s - V_{c_1}}{R_1} + \frac{V_s - V_{c_1} - V_{c_2}}{R_2} - \frac{V_{c_1} + V_{c_2}}{R_3}\right]$$

$$\frac{dV_{c_2}}{dt} = \frac{1}{C_2}\left[\frac{V_s - V_{c_1} - V_{c_2}}{R_2} - \frac{V_{c_1} + V_{c_2}}{R_3}\right]$$

$$\frac{d}{dt}\begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1}\left(-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}\right) & -\frac{1}{C_1}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \\ \frac{1}{C_2}\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) & \frac{1}{C_2}\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) \end{bmatrix} \cdot \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ \frac{1}{C_2 R_2} \end{bmatrix} V_s$$

OUTPUT = $V_{R_2} = V_s - V_{c_1} - V_{c_2}$

$$y = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} V_s$$