PROBLEM 6.3

(a) FROM Fig 6.26, FOR BALL-SCREW

$$V_n = n\,\Omega_n \qquad n = \frac{\text{rad/s}}{\text{m/s}} = \text{rad/m}$$

THEN FORCE ON NUT F_n IS related TO SHAFT TORQUE T_n BY

$$F_n = -\frac{1}{n} T_n$$

AND BALL-SCREW IS A TRANSFORMER WITH:

$$V_n \cdot F_n = n\Omega_n \left(-\frac{1}{n} T_n\right) = -\Omega_n T_n$$

$$V_n \cdot F_n + \Omega_n T_n = 0$$

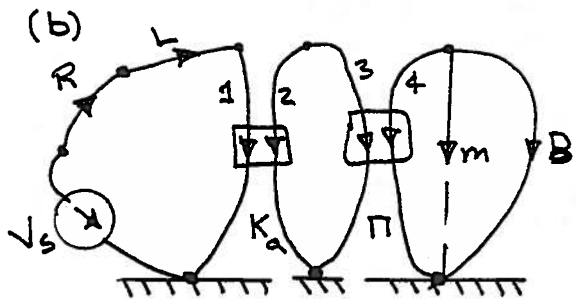
Now N : threads/cm

$\therefore \quad n = 200\pi N \quad \text{rad/m}$

ADVANCE ONE THREAD WITH ROTATION OF 2π rad

1 cm = .01 m

(b)



LINEAR GRAPH

STATE VAR

V_m, i_L

FOR DC MOTOR:

$$V_1 = K_a\,\Omega_2$$

$$i_1 = -{}^1/_{K_a}\,T_2$$

FOR BALL-SCREW:

$$F_4 = -\frac{1}{n} T_3$$

$$V_4 = n\,\Omega_3$$

PROBLEM 6.3 CONTINUED

$\frac{di_L}{dt} = \frac{1}{L} v_L$	$v_L = V_s - v_R - v_1$
$\frac{dv_m}{dt} = \frac{1}{m} F_m$	$F_m = -F_B - F_4$
$F_B = B\, v_B$	$v_B = v_m$
$v_R = R\, i_R$	$i_R = i_L$
$F_4 = -\frac{1}{n} T_3$	$T_3 = -T_2$
$\Omega_3 = \frac{1}{n} v_4$	$v_4 = v_m$
$T_2 = -K_a\, i_1$	$i_1 = i_L$
$v_1 = K_a\, \Omega_2$	$\Omega_2 = \Omega_3$

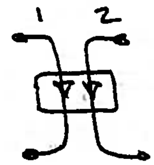
$$\frac{di_L}{dt} = \frac{1}{L}\left[V_s - R\,i_L - K_a\left(\frac{1}{n}\right) v_m\right]$$

$$\frac{dv_m}{dt} = \frac{1}{m}\left[-B v_m + \frac{K_a}{n}\, i_L\right]$$

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_m \end{bmatrix} = \begin{bmatrix} -R/L & -K_a/nL \\ K_a/nm & -B/m \end{bmatrix} \cdot \begin{bmatrix} i_L \\ v_m \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_s$$

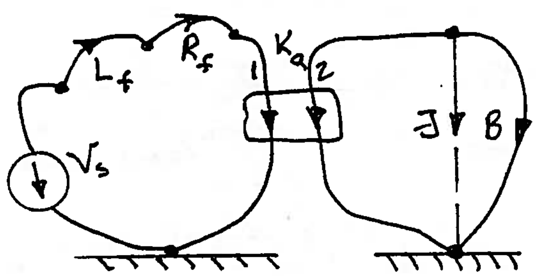
PROBLEM 6.7

(a) FOR MODEL USE A CONSISTENT dc motor REPRESENTATION WITH $P_1 + P_2 = 0$

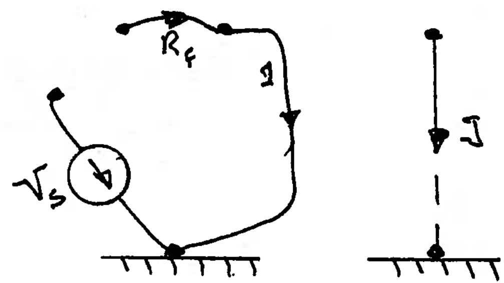


$$v_1 = K_a \Omega_2$$
$$i_1 = -\frac{1}{K_a} T_2$$

THE LINEAR GRAPH



NORMAL TREE



(b)

$\frac{d\Omega_J}{dt} = \frac{1}{J} T_J$	$T_J = -T_B - T_2$
$\frac{di_L}{dt} = \frac{1}{L_f} V_L$	$V_L = V_s - V_R - V_1$
$T_B = B\,\Omega_B$	$\Omega_B = \Omega_J$
$V_R = R_f\, i_R$	$i_R = i_L$
$v_1 = K_a \Omega_2$	$\Omega_2 = \Omega_J$
$T_2 = -K_a i_1$	$i_1 = i_L$

$$\frac{d\Omega_J}{dt} = \frac{1}{J}\left[-B\Omega_J + K_a i_L\right]$$

$$\frac{di_L}{dt} = \frac{1}{L}\left[V_s - R i_L - K_a \Omega_J\right]$$

$$\frac{d}{dt}\begin{bmatrix}\Omega_J \\ i_L\end{bmatrix} = \begin{bmatrix} -B/J & +K_a/J \\ -K_a/L & -R/L \end{bmatrix} \cdot \begin{bmatrix}\Omega_J \\ i_L\end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s$$

PROBLEM 6.7 CONTINUED

(c) MANUF. SPEC. WITH $T^* = K_m i$, $V = K_v \Omega$

K_m: OZ-IN/AMP $\quad K_v$: VOLTS/KRPM

FOR $K_m = 6$ OZ-IN/AMP

IN MANUF. CONVENTION $P_1 = P_2$ or $P_1 - P_2 = 0$

AND $T^* = -T$ AND SO

$$K_v = \frac{\Omega}{V} = K_a$$

$$K_m = \frac{T^*}{i} = \frac{-T}{i} = -(-K_a) = K_a$$

DETERMINE K_a IN SI UNITS

K_a: N-m/Amp : $6 \text{ OZ-IN/AMP} \ \frac{1 \text{ lb}}{16 \text{ OZ}} \ \frac{4.45 \text{ N}}{\text{lb}} \ \frac{1 \text{ ft}}{12 \text{ IN}} (.31) \frac{m}{ft}$

$$: \left(\frac{6}{16}\right)\left(\frac{4.45}{12}\right)(.31) \text{ N-m/AMP}$$

$$K_a = .043 \text{ N-m/AMP}$$

$$\therefore K_a = .043 \text{ V-s/r}$$

$$\text{rAD/s} \cdot 60 \frac{s}{m} \ \frac{1}{2\pi} \ \frac{1}{1000} = \text{KRPM}$$

$$\therefore K_v = \frac{.043}{60/(2\pi \cdot 1000)} \text{ V/KRPM} = 4.5 \text{ VOLTS/KRPM}$$

(d) WITH CONSTANT CURRENT $i_L = 1$ Amp

(i) $\frac{d\Omega}{dt} = 0 \quad \therefore \quad B\Omega_J = K_a i_L$ AND $B = \frac{.043(1)}{1000 \cdot 2\pi/60}$

or $B = 0.41 \cdot 10^{-3}$ N-m-s

(ii) $V_T = Ri + K_a \Omega = 3 + .043 \left(\frac{1000 \cdot 2\pi}{60}\right) = 3 + 4.5$ V $= 7.5$ V

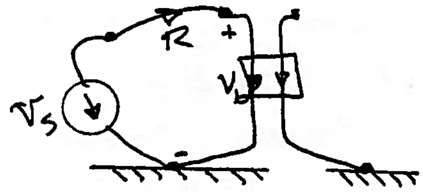
(iii) Total Power $= P_T = v \cdot i = 7.5$ W : RESISTOR, DAMPER

ELECT : $Ri^2 = 3$ W $\qquad$ MECH: $B\Omega^2 = 0.41 \left(\frac{1000 \cdot 2\pi}{60}\right)^2 10^{-3}$

$\%P_E = 40\%$ $\qquad$ $\%P_M = 60\%$ $\qquad = 4.492 \frac{\text{N-m}}{s} = 4.5$ W

PROBLEM 6.8

WITH dc MOTOR AT CONSTANT CONDITIONS INPUT CIRCUIT IS



$$V_s = iR + V_b$$

AND $i = \dfrac{V_s - V_b}{R}$ WITH $V_b = K_a \Omega$

NOW WHEN MOTOR STALLS $\Omega = 0$
$\therefore V_b = 0$ AND $i = V_s/R$ AND IS A MAXIMUM. HEAT DISSIPATED IN R IS also A MAX with $\Omega = 0$ AND IS i^2R

USING MOTOR DATA FROM PROBLEM 6.7
$i = V_s/R = 15/3 = 5A$ AND POWER DISSIPATED IN RESISTOR R IS: $P_R = 5^2 \cdot 3 = 75w$

REFERING TO PROBLEM 6.7 AT $\Omega = 1000$ rpm AND $V_s = 7.5v$, POWER DISSIPATED IN $R = 3w$

PROBLEM 6.9

(a) FOR COIL, EACH TURN HAS LENGTH $2\pi r$ AND FORCE F_T ON EACH TURN IS GENERATED DUE TO CURRENT i IN FIELD B:

$$F_T = -2\pi r B i$$

AND FOR N TURNS, TOTAL FORCE F

$$F = -2\pi r N B i$$

WHERE FROM FIG 6.30 F HAS NEGATIVE (−) SIGN

FOR VOLTAGE V_i IN COIL WITH VELOCITY v

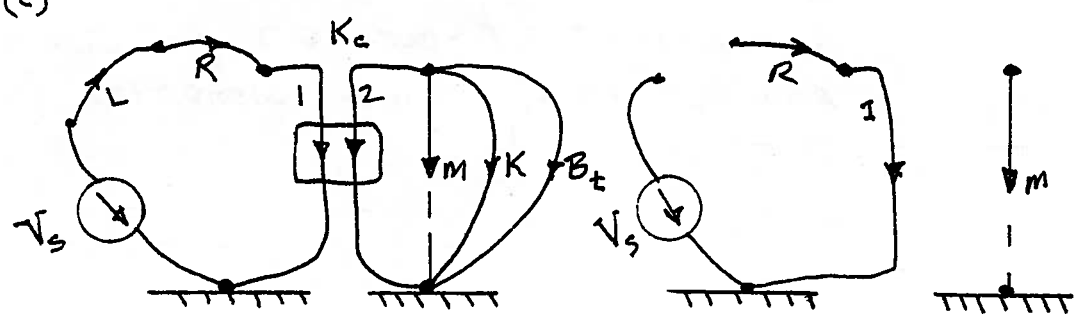
$$V_i = 2\pi r N B v$$

(b) AND RESULTING MODEL IS A TRANFORMER

$$\begin{bmatrix} V_i \\ i \end{bmatrix} = \begin{bmatrix} K_c & 0 \\ 0 & -1/K_c \end{bmatrix} \begin{bmatrix} v \\ F \end{bmatrix}$$

$$K_c = 2\pi r N B$$

(c)



LINEAR GRAPH NORMAL TREE

STATE VAR.: V_m, i_L, F_K

PROBLEM 6.9 CONTINUED

(d)

$\frac{dv_m}{dt} = \frac{1}{m} F_m$	$F_m = -F_2 - F_K - F_B$
$\frac{di_L}{dt} = \frac{1}{L} V_L$	$V_L = V_s - V_R - V_1$
$\frac{dF_K}{dt} = K\, v_K$	$v_K = v_m$
$F_B = B\, v_B$	$v_B = v_m$
$V_R = R\, i_R$	$i_R = i_L$
$F_2 = -K_c\, i_1$	$i_1 = i_L$
$V_1 = K_c\, v_2$	$v_2 = v_m$

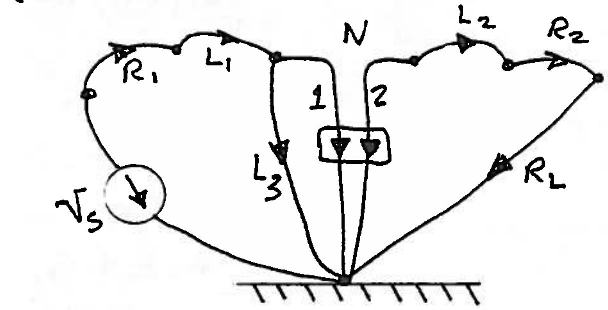
$$\frac{d}{dt}\begin{bmatrix} v_m \\ i_L \\ F_K \end{bmatrix} = \begin{bmatrix} -B/m & K_c/m & -1/m \\ -K_c/L & -R/L & 0 \\ K & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_m \\ i_L \\ F_K \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \\ 0 \end{bmatrix} V_s$$

OUTPUT $= v_m$

$$v_m = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_m \\ i_L \\ F_K \end{bmatrix}$$

PROBLEM 6.12

(a)



$N = N_2/N_1$

$V_2 = N\,V_1$

$i_2 = \frac{-1}{N}\,i_1$

NOTE: NORMAL TREE REQUIRES ONE INDUCTOR, SELECT L_3, It is ∴ dependent

$\frac{di_{L_1}}{dt} = \frac{1}{L_1} V_{L_1}$	$V_{L_1} = V_s - V_{R_1} - V_{L_3}$
$\frac{di_{L_2}}{dt} = \frac{1}{L_2} V_{L_2}$	$V_{L_2} = V_2 - V_{R_2} - V_{R_L}$
$V_{R_2} = R_2 i_{R_2}$	$i_{R_2} = i_{L_2}$
$V_{R_L} = R_L\, i_{R_L}$	$i_{R_L} = i_{L_2}$
$V_{R_1} = R_1\, i_{R_1}$	$i_{R_1} = i_{L_1}$
$V_2 = N\, V_1$	$V_1 = V_{L_3}$
$i_1 = -N i_2$	$i_2 = -i_{L_2}$
$V_{L_3} = L_3 \frac{di_{L_3}}{dt}$	$i_{L_3} = i_{L_1} - i_1$

$$(1)\quad \frac{di_{L_1}}{dt} = \frac{1}{L_1}\left[V_s - R_1 i_{L_1} - L_3\left(\frac{d}{dt} i_{L_1} - \frac{d}{dt}(+N i_{L_2})\right)\right]$$

$$(2)\quad \frac{di_{L_2}}{dt} = \frac{1}{L_2}\left[N L_3\left(\frac{di_{L_1}}{dt} - \frac{d}{dt}(+N i_{L_2})\right) - R_2 i_{L_2} - R_L i_{L_2}\right]$$

THE STATE EQ MAY BE WRITTEN:

$$(3)\quad \left(1+\frac{L_3}{L_1}\right)\frac{di_{L_1}}{dt} - \left(\frac{L_3 N}{L_1}\right)\frac{di_{L_2}}{dt} = \left(\frac{-R_1}{L_1}\right) i_{L_1} + \frac{1}{L_1} V_s$$

$$(4)\quad \left(-\frac{N L_3}{L_2}\right)\frac{di_{L_1}}{dt} + \left(1 + \frac{N^2 L_3}{L_2}\right)\frac{di_{L_2}}{dt} = -\frac{(R_2+R_L)}{L_2}\, i_{L_2}$$

PROBLEM 6.12 CONTINUED

AND PLACED IN A FORM TO SOLVE USING CRAMER'S RULE

$$\begin{bmatrix} \left(1+\frac{L_3}{L_1}\right) & -\frac{L_3 N}{L_1} \\ -\frac{N L_3}{L_2} & \left(1+\frac{N^2 L_3}{L_2}\right) \end{bmatrix} \begin{bmatrix} \frac{di_{L_1}}{dt} \\ \frac{di_{L_2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} i_{L_1} + \frac{V_S}{L_1} \\ -\frac{(R_2+R_L)}{L_2} i_{L_2} \end{bmatrix}$$

LET $D = \left(1+\frac{L_3}{L_1}\right)\left(1+\frac{N^2 L_3}{L_2}\right) - \frac{N^2 L_3^2}{L_1 L_2} = 1 + \frac{L_3}{L_1} + \frac{N^2 L_3}{L_2}$

THEN:

$$\frac{di_{L_1}}{dt} = \frac{1}{D}\left[\left(1+\frac{N^2 L_3}{L_2}\right)\left(-\frac{R_1}{L_1} i_{L_1} + \frac{V_S}{L_1}\right) - \frac{L_3}{L_1} N \frac{(R_2+R_L)}{L_2} i_{L_2}\right]$$

$$\frac{di_{L_2}}{dt} = \frac{1}{D}\left[-\left(1+\frac{L_3}{L_1}\right)\frac{(R_2+R_L)}{L_2} i_{L_2} + \frac{N L_3}{L_2}\left(-\frac{R_1}{L_1} i_{L_1} + \frac{V_S}{L_1}\right)\right]$$

AND

$$\frac{d}{dt}\begin{bmatrix} i_{L_1} \\ i_{L_2} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{D L_1}\left(1+\frac{N^2 L_3}{L_2}\right) & -\frac{L_3 N}{L_1 D L_2}(R_2+R_L) \\ -\frac{N L_3 R_1}{L_1 L_2 D} & -\frac{(R_2+R_L)}{L_2 D}\left(1+\frac{L_3}{L_1}\right) \end{bmatrix} \cdot \begin{bmatrix} i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{D L_1}\left(1+\frac{N^2 L_3}{L_2}\right) \\ \frac{N L_3}{D L_2 L_1} \end{bmatrix} V_S$$

(b) WITH V_S = constant, then i_{L_1} & i_{L_2} & i_{L_3} ARE ALSO CONSTANTS: REFER TO EQ 3 & 4 WHEN STATE VAR ARE CONSTANT, THEN

$$i_{L_1} = V_S/R_1 \qquad i_{L_2} = 0$$

SINCE $\frac{d}{dt} i_{L_1} = \frac{d}{dt} i_{L_2} = \frac{d}{dt} i_{L_3} = 0$

AND WITH $i_{L_2} = 0$ $\quad i_{R_L} = 0$ AND $V_{R_L} = 0$

Thus with constant INPUT, MODEL YIELDS 0 for OUTPUT – i.e. $V_{R_L} = 0$