1. Find the first five terms of the following.

(a) \(a_{n+1} = -3a_n + 4, \quad a_0 = 5\).

(b) \(x_{n+1} = \frac{2}{3x_n}, \quad x_0 = 1\).

(c) \(y_n = (-1)^n y_{n-1} + 5, \quad y_{-1} = 10\)

2. Determine whether the given sequence is a solution for the given recurrence relation.

(a) \(a_{n+1} = 3a_n - 2a_{n-1}, \quad a_n = 7\).

(b) \(x_{n+1} = -x_n + 2, \quad x_n = (-2)(-1)^n + 1\).

(c) \(y_{n+1} = 2y_n - 1, \quad y_n = 2 \cdot 2^n + 1\)

3. Write a recurrence relation for each of the following situations.

(a) A pod of whales has 32 members. The annual growth rate of the pod is 4%.

(b) You deposit $5000 in an account that pays an annual interest rate of 2.6% compounded monthly.

(c) Same as (b), but you make monthly deposits of $150.

Solve the recurrence relations with the specified initial conditions.

1. \(a_n = 5a_{n-1} - 6a_{n-2}, \quad n \geq 2, \quad a_0 = 1, \quad a_1 = 0\).

2. \(a_n = a_{n-1} + 6a_{n-2}, \quad n \geq 2, \quad a_0 = 3, \quad a_1 = 6\).

3. \(a_n = 7a_{n-1} - 10a_{n-2}, \quad n \geq 2, \quad a_0 = 2, \quad a_1 = 1\).

4. \(a_n = 6a_{n-1} - 8a_{n-2}, \quad n \geq 2, \quad a_0 = 4, \quad a_1 = 10\).

5. \(a_n = 4a_{n-2}, \quad n \geq 2, \quad a_0 = 0, \quad a_1 = 4\).

6. §1.8 #3a; do NOT do the cobweb diagram.

7. §1.8 #3b; do NOT do the cobweb diagram.

8. §1.8 #3c; do NOT do the cobweb diagram.
For #1 and #2, do the following:
(a) Generate the five-number summary.
(b) Draw a box-and-whiskers plot.
(c) Comment on the shape of the distribution (e.g., skewed or approximately symmetric).
(d) Identify outliers, if any.

1. \{21, 23, 24, 25, 29, 33, 49\}
2. \{10.2, 14.1, 14.4, 14.4, 14.5, 14.6, 14.7, 14.7, 14.9, 15.1, 15.9, 16.4\}

3. Consider the data set \{-24, 30, -42, 48, x\}. Identify a value of \(x\) such that
   (a) \(x\) is not an outlier.
   (b) \(x\) is an outlier such that it is too big but is within 10 of not being an outlier.
   (c) \(x\) is an outlier such that it is too small but is within 5 of not being an outlier.

4. §2.6 #1. You do not have to graphically present the data—but it might help your intuition.

5. §2.6 #4.

6. The following table presents the probability distribution of the number of defects \(X\) in a randomly chosen printed circuit board.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(x))</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Compute the mean number of defects.
(b) Compute the standard deviation.

1. The advisor of ASME at a large university believes the group consists of 10% freshmen, 20% sophomores, 40% juniors, and 30% seniors. The club has 14 freshmen, 19 sophomores, 51 juniors, and 16 seniors. Test the advisor’s hypothesis at \(\alpha = 0.10\).

2. In a nationwide survey of mechanical engineering majors to determine their area of specialization, 38% favored robotics, 32% chose aeronautics, 23% opted for manufacturing, and 7% expressed no preference. To determine if Washington state MEs followed the national trend, a local researcher queried 300 ME undergraduates in Washington. Of these, 122 choses robotics, 85 responded aeronautics, 76 said manufacturing, and 17 expressed no preference. At \(\alpha = 0.10\), test the claim that Washington state follows the national trend.

3. A staff member at an ambulance service wishes to determine whether the number of accidents is uniformly distributed throughout the week. A week was selected at random and the following data was obtained. At \(\alpha = 0.05\), test the claim that the number of accidents is uniformly distributed during a week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td># Accidents</td>
<td>28</td>
<td>32</td>
<td>15</td>
<td>14</td>
<td>38</td>
<td>43</td>
<td>19</td>
</tr>
</tbody>
</table>
1. Put the following matrices in the canonical form for absorbing chains.

(a) \[ P = \begin{pmatrix}
  s_1 & s_2 & s_3 \\
  1/3 & 1/3 & 1/3 \\
  0 & 1 & 0 \\
  1/3 & 1/2 & 1/6 \\
\end{pmatrix} \]

(b) \[ P = \begin{pmatrix}
  s_1 & s_2 & s_3 & s_4 \\
  1 & 0 & 0 & 0 \\
  1 & 0 & 0 & 0 \\
  1/4 & 1/4 & 1/4 & 1/4 \\
  0 & 0 & 0 & 1 \\
\end{pmatrix} \]

2. Write the absorbing Markov Chain in canonical form for the following random walk. States A, C, and D are absorbing, the others are transient. The probability of transitioning to an absorbing state is 10%; the probabilities of transitioning to a transient state are equal. For example, state \( F \) has a 10% chance of transitioning to absorbing state A and a 30% chance of transitioning to B, 30% to G, and 30% to J. Please put the transient states in alphabetical order!!

3. Write the absorbing Markov Chain in canonical form for the following random walk using the same diagram as the previous problem. This time, states B and E are absorbing. As before, the probability of transitioning to an absorbing state is 10%; the probabilities of transitioning to a transient state are equal.

4. Three tanks fight a duel by firing at the strongest opponent. Tank A hits its target with probability 2/3, tank B with probability 1/2, and tank C with probability 1/3. Shots are fired simultaneously, and once a tank is hit it is out of action. Write the absorbing Markov Chain in canonical form using the following states: A, B, C, AB, AC, BC, ABC, E. The letters represent the tanks that survive with E indicating the field is empty, i.e., all the tanks have been destroyed.

5. Modify the transition matrix in the previous exercise, assuming that when all tanks are in action, A fires at B, B at C, and C at A.
Solve the LP problems using the graphical/analytical procedure. You may check your answers with Excel but the work must be done with pencil/pen and paper.

1. Maximize $5x + 3y$ subject to
   \[
   \begin{align*}
   5x + 2y & \leq 40 \\
   3x + 6y & \leq 48 \\
   x & \leq 7 \\
   2x - y & \geq 3 \\
   x, y & \geq 0
   \end{align*}
   \]

2. Minimize $x + 2y$ subject to
   \[
   \begin{align*}
   x + 3y & \geq 90 \\
   8x + 2y & \geq 160 \\
   3x + 2y & \geq 120 \\
   y & \leq 70 \\
   x, y & \geq 0
   \end{align*}
   \]

3. Minimize $4x + 7y$ subject to
   \[
   \begin{align*}
   3x + 7y & \geq 231 \\
   10x + 2y & \geq 200 \\
   2y & \geq 45 \\
   2x & \leq 75 \\
   x, y & \geq 0
   \end{align*}
   \]

4. Maximize $7x + 4y$ subject to
   \[
   \begin{align*}
   9x + 8y & \leq 72 \\
   3x + 9y & \geq 27 \\
   9x - 15y & \geq 0 \\
   x, y & \geq 0
   \end{align*}
   \]

5. Tree Houses R Us (THRU) manufactures prefabricated tree houses (what a surprise!), large and small. Each large tree house requires 150 square feet of material while the small tree houses use 50 square feet. Each week the company buys 8,000 square feet of material. The large tree houses sell for $50 and the small tree houses sell for $20. THRU estimates that the company spends $1 in advertising for each tree house, regardless of size. The company has a weekly budget of $100 for advertising. The marketing gurus estimate that no more than 40 large tree houses can be sold in a week. Find the optimal number of tree houses to make to maximize revenue.