Practice test 2

Please e-mail me if you find errors in this or the solutions!

Instructions: I'd suggest trying this under test conditions.

1. Existence and Uniqueness theorem Perhaps: which of these DEs can we say have solutions? Are the solutions Unique? Where are they not Unique?
   $\frac{dy}{dt} = \frac{\sqrt{2-2y}}{\sqrt{2y-2}}$ $\quad \frac{dy}{dt} = \ln\left(\frac{1}{1+y^2} - 1\right)$ $\quad \frac{dy}{dt} = \ln\frac{1}{1+y^2} - 1$ $\quad \frac{dy}{dt} = \frac{t^2-4}{y^2-2y-3}$ $\quad \frac{dy}{dt} = \frac{y^2-2y-3}{t^2-4}$

2. The Existence and Uniqueness theorems both hold for $\frac{dy}{dt} = f(t,y)$ and y_1, y_2 and $y(t)$ all solve the differential equation what can we say about $y(t)$ as t goes to ∞ if: $y_1 = -1$ and $y_2 = \frac{1}{t-1} - 1$ and $y(0) = -\frac{3}{2}$?

3. Given that $\frac{dy}{dt} = 2\sqrt{|y|}$
   (a) Show that $y(t) = 0$ is a solution.
   (b) Find all of the solutions. [Hint: when people did this on the homework they broke it up, but didn't think carefully about integration constants.]
   (c) Why doesn't this differntial equation contradict the uniqueness theorem?

4. Find f_x, f_y and f_z for the following functions:
   $f(x,y,z) = x\sin(x+y)\cos(x+z)$ $\quad f(x,y,z) = e^{x^2+y^2}$

5. sketch the phase line and solutions for the following equations, label any equilibria as sources, sinks or nodes if they are neither a source nor a sink.
   $\frac{dy}{dt} = \cos^2 y$ $\quad \frac{dy}{dt} = (1-y^2)\sin(2y)$ $\quad \frac{dy}{dt} = y^3 - 8$ $\quad \frac{dy}{dt} = y + 1$
   When the arrows point out, it's a source, when they point in, it's a sink, when they are the same on either side, it's a node.

6. Given the following conditions draw the possibile phase lines, why are there more than one? Draw the corresponding possible graphs of $f(y)$.
   $\frac{dy}{dt} = 0$ for $y = 0$
   $\frac{dy}{dt} = 0$ for $y = 50$
   $\frac{dy}{dt} > 0$ for $100 > y > 50$
   $\frac{dy}{dt} < 0$ for $y > 100$

7. Identify which of the following equations are linear. Identify which are forced and which are unforced. $\frac{dy}{dt} = t^2$ $\quad \frac{dy}{dt} = y^2$ $\quad \frac{dy}{dt} = y^2t^2$ $\quad \frac{dy}{dt} = yt^2$ $\quad \frac{dy}{dt} = t^2\sin y + \cos t$

8. Justify the linearity principle.

9. Find the general solution for the following differential equation.
   $\frac{dy}{dt} = 7y + e^{6t}$ and $\frac{dy}{dt} = 7y + e^{7t}$

10. Make a good guess of what the particular solution would look like for:
    $\frac{dy}{dt} = y + e^t + e^{-t} + t\sin(2t)$

11. Use integrating factors to find the solutions to the following differential equations.
$\frac{dy}{dt} = -3\frac{y}{t} + 1$

12. Use integrating factors to find the solutions to the following differential equations with the given initial conditions.
$\frac{dy}{dt} = \frac{y}{t+1} + t^2 + t$ and $y(1) = 5$