

## Practice test 1

**Instructions:** I'd suggest trying this under test conditions. (No calculator, no notes) This is longer than the real test would be.

1. Simplify the following expressions

(a)  $\ln 16 - 2 \ln 4$

(b)  $2 \ln x^3 - \ln y^2 + \ln y^3 + 3 \ln y$

(c)  $e^{4 \ln x} - e^{3 \ln x}$

(d)  $\ln(e^{x^3}) - \ln(3e^x)$

2. Solve for  $y$ :

(a)  $2y^2 + y = t + 5$

(b)  $\frac{y-4}{y} = t^2 + 3$

(c)  $\frac{1}{y^2-1} = t^2 + 4$

(d)  $\ln y = -\ln 2t$

(e)  $\ln y = -\ln t + \ln 3t^2$

(f)  $\ln y - \ln(y+1) = t^2$

3. Find the derivative with respect to  $x$  of the following:

(a)  $\ln 2$

(b)  $e^{3x}$

(c)  $\frac{1}{3x}$

(d)  $\frac{1}{x^2}$

(e)  $\frac{1}{\sqrt{2x}}$

(f)  $\cos x$

(g)  $e^x \sin(2x)$

(h)  $\pi x$

(i)  $\frac{2}{1+\sin x}$

4. The next several exercises involve breaking apart a function into pieces that can be differentiated. Graph the function, and then look for the 'sharp points'. Find the derivative and then graph the derivative. Explain any discontinuities in the derivative.

(a)  $\sqrt{|x|}$

(b)  $|\cos x|$

5. Integrate both sides to find  $y(t)$ :

(a)  $\frac{dy}{dt} = \sqrt{t}$

(b)  $\frac{dy}{dt} = 2e^t$

(c)  $\frac{dy}{dt} = \frac{1}{t-1}$

(d)  $\frac{dy}{dt} = \frac{1}{t^2}$

(e)  $\frac{dy}{dt} = \sin(2t)$

(f)  $\frac{dy}{dt} = \sin t \cos t$

(g)  $\frac{dy}{dt} = \frac{1}{1+t^2}$

(h) Use partial fractions to find:  $\frac{dy}{dt} = \frac{3}{t(t-2)}$

(i) Use partial fractions to find:  $\frac{dy}{dt} = \frac{2}{(t-1)(t+2)}$

6. 5 fish are introduced to a lake devoid of any other fish. After 6 months the population has tripled, after one year there are 9x as many fish, and so on. Write a population model based on this information. Do you think that your model will be reasonable after 20 years?

7. You have a radioactive sample that loses 1/3 of its radioactivity in an hour. What's its half life?

8. You have a population of wombats in a given marsupial reserve. If there are fewer than 40 wombats they won't meet often enough to reproduce. But the area will only support 220 wombats. Design a logistic model that takes these parameters into account. Does this model have any equilibria?

9. Given the following predator-prey relationships:

$$\frac{dP}{dt} = \alpha P - \beta Pp \text{ and } \frac{dp}{dt} = \gamma p + \delta Pp$$

Assuming all parameters are positive. What do  $\alpha, \beta, \gamma, \delta, P$  and  $p$  represent (which is predator and which is prey)?

10. You have the following differential equation  $\frac{dy}{dt} = y^3 + y^2 - 2y$ .

(a) For what values of  $y$  is  $y(t)$  in equilibrium?

(b) For what values of  $y$  is  $y(t)$  decreasing?

(c) For what values of  $y$  is  $y(t)$  increasing?