The idea of a Taylor series

These exercises will hopefully take you through the idea of a Taylor series. We're going to start with the first order Taylor Series, which is the same as the linear approximation:
$L(x) = f(x_o) + \frac{df}{dx}_{x_o}(x - x_o)$
and develop this into the higher order terms:
$f(x) \approx f(x_o) + \frac{df}{dx}(x_o)\frac{(x-x_o)}{1!} + \frac{d^2f}{dx^2}(x_o)\frac{(x-x_o)^2}{2!} + \frac{d^3f}{dx^3}(x_o)\frac{(x-x_o)^3}{3!} + \cdots = \sum \frac{d^nf}{dx^n}(x_o)\frac{(x-x_o)^n}{n!}$

1. Find the linear approximation of sin$x$ for $x = 0$.

2. Check to see how well this works for sin 1, sin 0.1, sin 0.01, and sin0.001, where 0.001 is in radians. Does it work better or worse as the angle gets smaller?

3. If we have a large angle we need to add higher order terms. The next term goes as the second derivative:
   $f(x) \approx f(x_o) + \frac{df}{dx}(x_o)\frac{(x-x_o)}{1!} + \frac{d^2f}{dx^2}(x_o)\frac{(x-x_o)^2}{2!}$
   What's the next order term in this case?

4. The last problem gave us the quadratic term, the cubic term will stay in the equation:
   $f(x) \approx f(x_o) + \frac{df}{dx}(x_o)\frac{(x-x_o)}{1!} + \frac{d^2f}{dx^2}(x_o)\frac{(x-x_o)^2}{2!} + \frac{d^3f}{dx^3}(x_o)\frac{(x-x_o)^3}{3!}$
   Write out the x^3 term.

5. What do you notice about all of the even terms for sin x? Why might the term 'odd function' be appropriate?

6. Write down the first 4 non-vanishing (non-zero) terms for sin x.

7. Now, let's try cos x, what's the linear approximation?

8. Now write out the first 4 non-vanishing (non-zero) terms? Why might the term 'even function' be appropriate?

9. Now, let's try the exponential function. Write down a linear approximation of the exponential function.

10. Write down the first 4 non-vanishing (non-zero) terms? Is this either an even or an odd function, or does it have both even and odd exponents?