Homework 7

Chapter 2: Figure out which variables represent preditor and prey in the following equations then do Problems 1-6 for the following set of equations (as opposed to the ones given in the book):
i. $\frac{dU}{dt} = -0.1U^2 + U + 113UW \qquad \frac{dW}{dt} = -15W^2 + 1300W - 0.1UW$
and
ii. $\frac{dU}{dt} = -3U^2 + U + 0.2UW \qquad \frac{dW}{dt} = -\frac{W^2}{8} + 3W - 10UW$

Also from Chapter 2: 8-11, 22, 24, 28
Additionally not from Chapter 2, but the same material:

1. You have a 2kg mass on a spring with spring constant $k = 20\frac{N}{m}$ attached to a wall (this is a horizontal mass-spring system, as opposed to what's in the book). At $t = 0$ the mass is given an initial velocity of $0.5\frac{m}{s}$ to the right (so that the spring is extending).
   (a) Draw a picture of this system and it's initial condition.
   (b) Using Newton's second law and Hooke's law, write down a 2^{nd} order differential equation for this system.
   (c) Come up with an expression of the kinetic energy of this system as a function of time. (Recall: KE=$\frac{1}{2}mv^2$).
   (d) Come up with an expression of the potential energy of this system as a function of time. (Recall: PE=$\frac{1}{2}kx^2$)
   (e) Show that the energy of this system does not depend on time.
2. You have a circuit with a switch, a capacitor and an inductor in series (and nothing else but the wire connecting the circuit). The capacitor has capacitance C and the inductor has inductance L.
   (a) Draw this circuit.
   (b) Using Kirchoff's voltage law (the sum of the voltages around a closed circuit is zero) find a differential equation for the circuit (once the switch is closed) given that the voltage across a capictor is: $V_c = \frac{q}{C}$ and the voltage across an inductor is: $V_L = L\frac{di}{dt}$.
   (c) Electrical current, i is simply how much the charge changes with time, express this in a differential equation.
   (d) Write your set of 2 differential equations as one single second order differential equation.
   (e) It's often said that this system strongly resembles the mass on a spring system. What do $V, L, q, i, \frac{1}{C}$ and t correspond to in the mass on a spring system? What's the resonant frequency of the mass-spring system? What's the resonant frequency of this circuit?
   (f) This type of correlation is often useful because if we solve one system, then we have the solution for a whole host of related problems. Given the solution we had to Exercises 19 and 20 in class (or the previous question on this assignment), what's the equation for voltage and current for our circuit if $V_c(0) = 3V$, $C = 5\mu F$, $L = 15H$, and $i(0) = 0$?