

Mixing Problem

Since we already know how to solve these equations, I'm just setting them up, since that seems to be the part that people don't know how to do.

- Consider a pond that has 5 million gallons of fresh water, initially. If fish add $0.5 \frac{\text{ton}}{\text{year}}$ of fecal matter into the pond, and algae process $0.4 \frac{\text{ton}}{\text{year}}$ of the waste product. 100,000 gallons of fresh water flows in; the same rate that the mixture flows out. Design a differential equation that shows how the total amount, Q , of fish waste as a function of time.

$$\frac{dQ}{dt} = 0.5 \frac{\text{tons}}{\text{year}} - 0.4 \frac{\text{tons}}{\text{year}} - 100,000 \frac{\text{gal}(\text{pond})}{\text{year}} C$$

$$\frac{dQ}{dt} = 0.1 \frac{\text{tons}}{\text{year}} - 100,000 \frac{\text{gal}(\text{pond})}{\text{year}} \frac{S}{1,000,000} = \frac{dQ}{dt} = 0.1 - 0.1S$$

- If smoke from a single car in a garage is giving off $9\mu\text{g}$ of smoke every second in a room with 1500m^3 of air, what's the differential equation showing the concentration of smoke in the room?

$$\frac{dC}{dt} = \frac{9 \frac{\mu\text{g}}{\text{s}}}{1500\text{m}^3} = 0.006 \frac{\mu\text{g}}{\text{sm}^3}$$

$$\text{The total amount of smoke would be: } \frac{dS}{dt} = 9 \frac{\mu\text{g}}{\text{s}}$$

- Same as before, but now add a crossbreeze moving 3m^3 of fresh air into the room and the same amount of air out of the room.

$$\frac{dC}{dt} = \frac{9 \frac{\mu\text{g}}{\text{s}}}{1500\text{m}^3} - \frac{3\text{m}^3}{1500\text{m}^3} C = 0.006 \frac{\mu\text{g}}{\text{sm}^3} - 0.002C$$

$$\text{The total amount of smoke would be: } \frac{dS}{dt} = 9 \frac{\mu\text{g}}{\text{s}} - 3\text{m}^3 C = 9 \frac{\mu\text{g}}{\text{s}} - 3\text{m}^3 \frac{S}{1500\text{m}^3} = 9 \frac{\mu\text{g}}{\text{s}} - 0.002S$$