Partial Fractions

So: Here are some partial fractions:

- $\frac{3}{(y+1)(y+2)}$
- $\frac{4}{(2y+1)(y-2)}$
- $\frac{1}{(3y)(y+1)}$
  So we have a bit of a recipe here. We take our fraction, and the point is to break it into the sum of other fractions. So in general:
  $\frac{stuff}{f(y)g(y)} = \frac{A}{f(y)} + \frac{B}{g(y)}$
  Cross multiply to get: $\frac{stuff}{f(y)g(y)} = \frac{Ag(y)+Bf(y)}{f(y)g(y)}$
  Since the bottoms are the same we just look at the tops: $stuff = Ag(y) + Bf(y)$ Then we go through and seperate out the y from the constants (since they're linearly independent. We can change our value for y, but this whole expression must still be true, so, we must have A and B that hold true for all values of y, we create two equations (more if there are y^2, but that won't be on the test. So, specifically:

- $\frac{3}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+3} = \frac{A(y+3)+B(y+1)}{(y+1)(y+3)}$
  Note that the cross multiplication means we switch which functions are associated with A and B, and now the bottoms are the same on both sides. So we just take the numerators.
  $3 = A(y+3) + B(y+1) = Ay + 3A + By + B = (Ay + By) + 3A + B$
  $Ay + BY = 0 \quad 3 = 3A + B$ Since there are no ys on the left hand side the two terms with y in them on the right hand side must go to zero. Therefore: $A = -B$. Plugging this in to the right hand equation we get: $3 = 3A - A = 2A \quad A = \frac{3}{2} \quad B = -\frac{3}{2}$. This means that our original expression can be written as: $\frac{3}{(y+1)(y+2)} = \frac{3}{2(y+1)} - 32(y+2)$
  And if you put those two fractions over one common denomenator, you should get the left hand side! (Which is the whole point of partial fractions.)

- $\frac{4}{(2y+1)(y-2)} = \frac{A}{2y+1} + \frac{B}{y-2} = \frac{A(y-2)+B(2y+1)}{(2y+1)(y-2)}$
  $4 = A(y-2) + B(2y+1) = Ay - 2A + 2By + B = (A+2B)y + (B-2A)$
  $A+2B = 0 \quad 4 = B-2A \quad 4 = B+4B = 5B \quad B = \frac{4}{5} \quad A = -\frac{8}{5}$ $\frac{4}{(2y+1)(y-2)} = -\frac{8}{5(2y+1)} + \frac{4}{5(y-2)}$

- $\frac{1}{(3y)(y+1)} = \frac{A}{3y} + \frac{B}{y+1}$
  $1 = Ay + A + 3yB \quad A + 3B = 0 \quad A = 1 \quad B = -\frac{1}{3}$
  $\frac{1}{(3y)(y+1)} = \frac{1}{3y} - \frac{1}{3(y+1)}$