

Practice test 4

Please e-mail me if you find errors in this or the solutions!

Instructions: I'd suggest trying this under test conditions

- You have two different predators that both eat the same prey. Differential equations for the two sets of predator prey relations are provided (each presumes that there is only one predator eating the prey).
 - $\frac{dR}{dt} = 0.2R - 3RF$ $\frac{dF}{dt} = -0.1F + 2FR$
 - $\frac{dR}{dt} = 0.2R - 0.3RL$ $\frac{dL}{dt} = -2L + 0.3LR$
 - Why is the coefficient of R in both $\frac{dR}{dt}$ equations the same value (0.2)?
 - One of the predators is very active and needs a lot of food, and one is very laid back and sleeps a lot, assuming both are roughly the same size animals, which is which?
 - Find the equilibrium populations for i. and ii. separately.
 - If the population of R starts off at 1 and the population of L starts off at 1, how do the populations change at this point?
 - If the population of R starts off at 1 and the population of F starts off at 1, how do the populations change at this point?
 - If you want to have an equation for $\frac{dR}{dt}$ that includes both what would it be?
 - Would your equilibrium values in the separate cases work for the combined equation? If yes, why? If no, what would you need to change?
- Given $\frac{d^2y}{dt^2} + \frac{b}{m} \frac{dy}{dt} + \frac{k}{m}y = 0$
 - Break this into two linear first order differential equations.
 - Find the equilibrium solution to these two linear first order differential equations.
 - Using the s method find the solution to the differential equation.
 - The s method gives an exponential solution, but we also know that sinusoidal solutions are also possible. How do we generate sinusoidal solutions to this differential equation? What type of relationship between b , m and k will give these sinusoidal solutions? Mechanically, what do we call these different solutions?
- Consider $\frac{d^2y}{dt^2} + 4y = 0$ with $y(0) = 0$ and $y'(0) = v(0) = 2$
 - Find $y(t)$ with the given initial conditions.
 - Graph the initial condition on a $y - v$ graph (where y is on the horizontal axis and v is on the vertical). And label this point 'I'.
 - Graph the solution to the differential equation, include the direction the solution goes.
 - A little damping is added, mathematically what does this look like for the differential equation?
 - Draw what the graph would look like now with a little bit of damping.

- (f) Now, instead of adding a little damping, add a lot of damping, what does the graph look like now?
4. Given $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ derive that the period of oscillation for this system is $T = 2\pi\sqrt{LC}$
5. Given $A + 2B \rightarrow C$ and $A + C \rightarrow D$
- What's the $\frac{dA}{dt}$?
 - What's the $\frac{dB}{dt}$?
 - What's the $\frac{dC}{dt}$?
 - What's the $\frac{dD}{dt}$?
 - If you start with no C or D will you ever get any D ?
6. Determine the vector field for $\frac{dx}{dt} = x$ $\frac{dy}{dt} = -y$.
7. Given the picture on the website draw x vs. t and y vs. t for A B and C.
8. Is $\mathbf{Y}(t)$ a solution to $\frac{dx}{dt} = 3x + 3y$ $\frac{dy}{dt} = 2x + 4y$
- $\mathbf{Y}(t) = (3e^t, -2e^t)$
 - $\mathbf{Y}(t) = (-3e^t, 2e^t)$
9. Find $\mathbf{Y}(t)$ the solution to the set of differential equations: $\frac{dx}{dt} = 3x + 6y$ $\frac{dy}{dt} = 3y$
10. Given that if you graph the critically damped solution you get an exponential decay:
- Graph the critically damped case and the overdamped case, label which is which.
 - In a separate graph, graph the critically damped case and the underdamped case, label which is which.